Comments on 'Systematics of radial and angular-momentum Regge trajectories of light non-strange $q\bar{q}$ states', Masjuan, Arriola and Broniowski

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Abstract

These authors claim that the slope of the light-quark radial Regge trajectories is 1.35 ± 0.04 GeV^{-2}, disagreeing with the Crystal Barrel value 1.143 ± 0.013 GeV^{-2}. There are weaknesses in their choice of data. When these are corrected, results come back close to the Crystal Barrel average for the slope. A revised average value is 1.136 ± 0.012 GeV^{-2}.

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1 Introduction

Masjuan, Arriola and Broniowski (referred to later as MAB for brevity) report a new analysis of the slopes of Regge trajectories [1] [2]. They adopt a χ^2 criterion which inflates the errors assigned to resonance masses by up to a factor 20. They take

$$\chi^2 = \sum_n \left(\frac{M_n^2 - M_{n,exp}^2}{\Gamma_n M_n} \right)^2 , \tag{1}$$

where M_n and Γ_n are masses and widths of fitted resonances. Their rationale is that the extrapolation to poles off the real s-axis may be inaccurate by one half-width.

The essential point of disagreement with MAB is whether this assumption is justified or not. My point is that large uncertainties in the extrapolation to the pole arise only where a strong S-wave threshold opens in the immediate vicinity of the resonance. An example is $f_2(1565)$ which decays strongly to the $\omega\omega$ S-wave, opening precisely at 1565 MeV. In this case, there is a large dispersive contribution to the real part of the amplitude, as discussed below in Section 3. Clearly one should be alert to such thresholds. However, most high mass resonances have many open channels and such effects are small. After eliminating the special cases, there is no support for the assumption of Eq. (1) as the general case.

To illustrate the effect of their inflated errors, it is sufficient to quote just one example. The PDG quotes a mass for the $a_4(2040)$ as 2001 ± 10 MeV [3]. The experiments they use for this average have several final states and cover masses 1944 ± 50 to 2040 ± 30 MeV. For $a_4(2255)$ the PDG uses two measurements with masses 2237 ± 5 and 2255 ± 40 MeV. From these two states, MAB find a slope of 1.0 ± 0.8. This implies an error in the mass difference of 203 MeV. However, with this error, two neighbouring states are not resolved experimentally. This is entirely inconsistent with the Crystal Barrel assessment of errors.

Further disagreements arise from several sources. Firstly, there are some missing states. In particular, the spectroscopy of ρ and ω states, where both $^3S_1$ and $^3D_1$ states are expected, is in poor shape at present. A second point is that it is well known that $c\bar{c}$ and $b\bar{b}$ $^3S_1$ ground states are anomalously low in mass compared with their radial excitations. There are indications that this is also true for $n\bar{n}$ states. A third point is that there is almost certainly a $J^{PC} = 0^{++}$

glueball in the mass range 1370 to 1800 MeV, but no present agreement on its identification. It will certainly mix with $q\bar{q}$ states, and this mixing is likely to shift their masses unpredictably.

A problem is that MAB do not distinguish clearly between $^3S_1$ and $^3D_1$ states. Conventional wisdom is that P-state light mesons appear at masses 1200–1300 MeV, D states at 1600–1700 MeV and F states near 2000 MeV. However, they assign the third $^3S_1$ ω and ρ states masses of ~ 1970 MeV, i.e. ~ 300 MeV above the $^3D_3$ states $\omega_3(1670)$ and $\rho_3(1690)$. The result is a slope for the $^3S_1$ ρ and ω trajectories $\sim 4/3$ times larger than other trajectories.

They replace the best determinations of trajectories for $I = 0$, $C = +1$ mesons (where there are 10 sets of data) by poorer determinations of $I = 1$ $C = +1$ mesons, where there are no polarisation data to separate $^3P_2$ and $^3F_2$ mesons, hence much larger errors for masses. That biases their conclusions severely. An isolated point is that they replace the Crystal Barrel determination of the mass of $f_3(2300)$ by including a possibly biased mass determination from data on $\bar{p}p \to \Lambda\bar{\Lambda}$. That is not a good idea, since mixing with the $s\bar{s}$ amplitude can confuse the situation.

In order to present the discrepancies with the slopes of trajectories assigned by Crystal Barrel (hereafter abbreviated to CB), the slopes of all trajectories are redetermined from final CB data sets and tabulated for comparison with the slopes of MAB. The table of results makes the differences immediately apparent.

2 Prologue

The Crystal Barrel has produced extensive data on formation of high mass mesons in the process $\bar{p}p \to R \to A + B$, where R stands for a resonance and there are 18 channels of all-neutral final states available. Ref. [4] reviews the data and technical details. The detector covers 98% of the solid angle with CsI crystals which measure all-neutral final states. Quantum numbers fall into four non-interfering families $I = 0$ or 1, $C = +1$ or -1.

For $I = 0$, $C = +1$, there are data on 6 channels: $\eta\pi^0\pi^0$, $\eta'\pi^0\pi^0$, 3η, $\pi^0\pi^0$, $\eta\eta$ and $\eta\eta'$. There are also differential cross sections and polarisation data for $\bar{p}p \to \pi^+\pi^-$ from PS172 [5] and an earlier experiment at the CERN PS [6]. These are of vital for two reasons. Firstly, they separate $^3P_2$ and $^3F_2$ states. Secondly, polarisation is phase sensitive and reduces errors on fitted masses and widths substantially. The improvement in mass determination from polarisation data can be up to a factor 4, because of its phase sensitivity.

In the $\eta\pi\pi$ data there are prominent $f_4(2050)$ and $f_4(2300)$ signals, easily identified from their strong angular dependence. They are determined accurately in mass and width from data at 9 beam momenta from 600 to 1940 MeV/c. These states serve as interferometers for all lower spin triplet partial waves. There is also a lucky break, that two singlet states also appear prominently: an $\eta_2(2250)$ in $\eta'\pi\pi$ and $\eta(2320)$ in the 3η data in the channel $f_0(1500)\eta$. A complete set of $q\bar{q}$ states appears in two towers of resonances centred near 2000 and 2270 MeV. For $I = 1$, $C = -1$, an almost complete set of states also appears, but with poor identification of $^3S_1$ states. For $I = 1$, $C = +1$, there are actually two solutions, with one of them close to the $I = 0$, $C = +1$ solution, as one would expect for light quarks with small mass differences. For $I = 0$, $C = -1$, statistics are low for $\omega\eta$ and the $\omega\pi^0\pi^0$ data have the problem that the broad $\sigma \equiv f_1(500)$ interferes all over the Dalitz plot.

The F states lie systematically ~ 70 MeV above the P states, because high L states need to

overcome a centrifugal barrier in order to resonate. The D states lie roughly midway; S and G states continue the sequence.

There is a further essential source of information concerning the separation of $n\bar{n}$ and $s\bar{s}$ mesons. The $f_2(1525)$ is widely accepted as the $s\bar{s}$ partner of $f_2(1270)$. Production of $f_2(1525)$ in the Crystal Barrel experiment is extremely weak. It is detected at the 1–2% level in $\bar{p}p \to \eta\eta\pi^0$ in flight [7]. The conclusion is that $\bar{p}p$ annihilation is dominantly to $n\bar{n}$ final states - hardly a surprise. This conclusion is supported and quantified by a combined analysis of data on $\bar{p}p \to \pi^+\pi^-$, $\pi^0\pi^0$, $\eta\eta$ and $\eta\eta'$ [8]. Amplitudes for decay to $\eta\eta$ and $\eta\eta'$ depend on the well known composition of η and η' in terms of singlet and octet states and the pseudoscalar mixing angle. The observed state R is expressed as a linear composition $R = \cos\Phi|q\bar{q}> + \sin\Phi|s\bar{s}>$. The result is that $\Phi \leq 15^\circ$, i.e. a maximum of 25% in amplitude, for all observed states with the exception of $f_0(2105)$ (which is taken as a glueball candidate, but could possibly be due to unexpectedly strong mixing between $n\bar{n}$ and $s\bar{s}$). The allocation MAB make between $n\bar{n}$ and $s\bar{s}$ states is in conflict with the fact that CB states are dominantly $n\bar{n}$.

The partial wave analysis of CB data is documented in Section 4 of Ref. [4] This describes systematic checks which were made on the identification of resonances, particularly their stability as the number of fitted resonances was changed. Following sections illustrate results and discuss individual resonances and their Argand diagrams. For $I = 0$ $C = +1$, all states have statistical significance > 25 standard deviations except for the $f_2(2001)$ which is 18σ but observed clearly in 4 sets of data. Two states, $f_1(2310)$ and $\eta(2010)$ have rather large errors for masses. Regge trajectories are discussed in Section 9. For channels $\omega\pi$ and $\omega\eta$, polarisations of ω are determined by the angular dependence of decays to $\pi^+\pi^-\pi^0$ and are very revealing. The interpretation of this polarisation is important and discussed in Sections 7.1 and 8. There is one non-standard piece of nomenclature. These polarisations are described as vector polarisation P_y. Strictly, the standard nomenclature is that this should be called $\mathrm{Re\,iT}_{11}$, where T is tensor polarisation.

3 Determination of slopes of trajectories

One should be aware in advance that states may deviate from straight trajectories because of dispersive effects on resonance masses. The strict form for the denominator of a Breit-Wigner amplitude is

$$D(s) = M^2 - s - m(s) - i\sum_j g_j^2\rho_j \tag{2}$$

$$m(s) = \frac{1}{\pi}P\int_{sthr}^{\infty}\sum_j \frac{g_j^2\rho_j(s')\,ds'}{s'-s}. \tag{3}$$

To make the Principal Value integral converge better, it is usual to make a subtraction on resonance, though in principle this can be done at any mass; $sthr$ is the s value at threshold for each channel. The g_j^2 are coupling constants to every decay channel, and $\rho_j(s')$ are the phase space for each final state, including centrifugal barriers and form factors. Near the thresholds of important decay channels, a change in the imaginary part of the amplitude is accompanied by a corresponding real part so as to obey analyticity. At sharp thresholds, a cusp in the real part acts as an attractor [9]. The $a_0(980)$ and $f_0(980)$ are attracted to the $K\bar{K}$ S-wave threshold, and likewise the $f_2(1565)$ is attracted to the sharp threshold for the $\omega\omega$ final state; this attraction is

augmented by a broader threshold in the $\rho\rho$ channel, which has 3 times more events from the SU(2) relation with $\omega\omega$. For CB data in flight, there is a large number of open decay channels and such effects are blurred out, but they may still be present at a low level. There is no evidence for large effects in the mass range above 1950 MeV, though the $\bar{p}p$ threshold itself could act as an attractor.

Each set of quantum numbers will be examined one by one, fitting the expected states to observed masses and errors, but using common sense where states are missing or strongly displaced by dispersive effects. Having done this, a grand average is taken of all slopes. As a guide, Fig. 1 shows the updated analysis of the $I = 0$, $C = +1$ states.

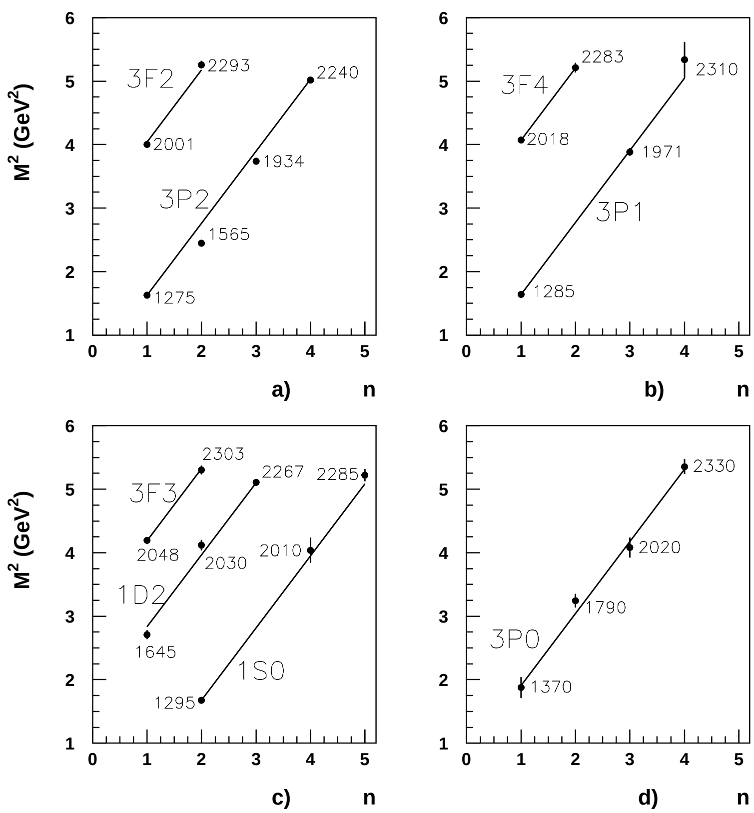

Figure 1: Trajectories of light mesons with $I = 0$, $C = +1$ observed in Crystal Barrel data in flight, plotted against radial excitation number n; masses are marked in MeV.

4 Results

Table 1 lists slopes for all families of light mesons. General comments are as follows. Firstly, $I = 0$, $C = +1$ states are best determined, because of the available polarisation data, which are very precise. For triplet states, decays are possible for orbital angular momenta $L = J + 1$ and $L = J - 1$. The ratio of coupling constants $r_J = g_{J+1}/g_{J-1}$ is tabulated in Ref. [4]; this is an important guide to whether states have $L = J + 1$ or $J - 1$.

Family	Resonance masses	CB slope	MAB slope	Comments
f_4	2050,2300	1.140 ± 0.093	-	
f_3	2050,2300	1.147 ± 0.071	1.27 ± 0.64	+
$f_2(^3F_2)$	2000,2295	1.254 ± 0.075	-	
$f_2(^3P_2)$	1270,1565,1910,2240	1.113 ± 0.025	-	+
f_1	1285, – ,1970,2310	1.130 ± 0.064	1.19 ± 0.15	+
f_0	1370,–,–,2330	1.24 ± 0.045	1.24 ± 0.18	+
h_3	2025,2275	1.075 ± 0.147	1.08 ± 0.54	
h_1	1170,1595,1965,2215	1.195 ± 0.059	1.20 ± 0.25	
b_3	2025,2275	0.95 ± 0.191	1.08 ± 0.54	
b_1	1235,-,1960,2240	1.169 ± 0.058	1.17 ± 0.18	
ω_3	1670,1945,2255	1.059 ± 0.163	1.16 ± 0.26	
ω_2	1975,2195	0.917 ± 0.160	-	
$\omega_1(^3D_1)$	1670,1960,2290	1.091 ± 0.068	1.27 ± 0.47	+
$\omega_1(^3S_1)$	782,1420,1650,–,2205	1.080 ± 0.029	1.50 ± 0.12	+
a_4	2040,2255	1.000 ± 0.045	1.00 ± 0.8	
a_3	2030,2275	1.051 ± 0.170	1.5 ± 1.1	
$a_2(^3F_2)$	2030,2255	0.964 ± 0.128	1.00 ± 0.7	
$a_2(^3P_2)$	1320,1700,1950,2175	1.000 ± 0.060	1.39 ± 0.26	+
a_1	1260,1640,1930,2270	1.084 ± 0.063	1.36 ± 0.49	+
a_0	1450,2025	0.964 ± 0.073	1.42 ± 0.26	+
π_2	1670,2005,2285	1.218 ± 0.062	1.21 ± 0.36	
π_0	1300,1800?,2070,2360	1.29 ± 0.200	1.27 ± 0.27	
ρ_3	1690,1990,2265	1.094 ± 0.050	1.19 ± 0.32	+
ρ_2	1940,2225	1.206 ± 0.085	-	
$\rho_1(^3D_1)$	1700,2000,2270	1.203 ± 0.052	1.08 ± 0.47	+
$\rho_1(^3S_1)$	770,1450,–,1900,2150	1.365 ± 0.108	1.43 ± 0.13	+
η_2	1645,2030,2265	1.188 ± 0.038	1.32 ± 0.32	
η_0	1295,2320	1.241 ± 0.030	1.33 ± 0.11	+

Table 1: Nominal resonance masses and slopes from CB and MAB analyses; comments are marked by a + in the final column and discussed in the text; a dash in column 2 indicates a missing state (or unused for reasons discussed in the text).

Why should $^3F_2$ $n\bar{n}$ states decay preferentially to $\bar{p}p$ $^3F_2$? A feature of CB data is that F states decay strongly to channels with high angular momentum. The origin of this is clearly a good overlap between wave functions of initial and final states. Llanes-Estrada et al. point out a formal analogy with the Frank-Condon principle of molecular physics consistent with this interpretation [10].

It is clear from the errors in the table that the χ^2 weighting used by MAB inflates some errors by large amounts. From the agreement in many cases between CB and MAB, it is also clear that remaining discrepancies should be inspected closely. It is easiest to compare with results from MAB in their Section III, taking them in the reverse order to the publication, i.e. K to A. A clear picture of disagreements then arises step by step.

The f_1 and f_3 states are considered in MAB Section K. The CB approach is to fit $f_1(1285)$, $f_1(1910)$ and $f_1(2310)$ using their errors. The first two have small errors, with the result that the fit misses the weak $f_1(2310)$ by just over one standard deviation. Neither $f_1(1420)$ nor $f_1(1510)$ is fitted well by the CB approach or that of MAB. In the CB fit, a state is expected at 1660 MeV, close to its isospin partner $a_1(1640)$. The $f_1(1510)$ is naturally assigned as the $s\bar{s}$ analogue of $f_1(1285)$; the mass difference is close to that between $f_2(1270)$ and $f_2(1525)$. Longacre proposes that $f_1(1420)$ is a molecule where an $L = 1$ pion circles a $K\bar{K}$ core [11]. For f_3, the MAB slope has an error a factor 9 larger than the CB value. Sections J and I agree well on slopes between CB and MAB for b_3, b_1, h_3 and h_1 states.

Section H considers ω states. There is agreement between MAB and CB on the ω_3 trajectory within errors. But then they put the $\omega(1650)$ on a trajectory with an $\omega(2290)$ observed in $\bar{p}p \to \Lambda\bar{\Lambda}$. The latter could however be an $s\bar{s}$ state or could simply be due to an upward shift of the $\omega(2255)$ because of mixing non-resonant background from $s\bar{s}$. The consequence is that they ignore the well known $\omega(1650)$ as a standard ${}^3D_1$ state.

The conventional scheme is to take the $\omega(1420)$ as a ${}^3S_1$ state and $\omega(1670)$ as ${}^3D_1$. The ρ states run parallel, with $\rho(1450)$ as a ${}^3S_1$ state and the well established $\rho_3(1690)$ as ${}^3D_1$. The $\phi(1680)$ fits in as ${}^3S_1$ and $\phi(1850)$ as ${}^3D_1$. The ϕ states lie higher than $n\bar{n}$ states by a similar mass difference to that between $f_2(1270)$ and $f_2(1525)$ (which is well established as $s\bar{s}$). Fitting the $\omega(1960)$ together with $\omega(1650)$ gives a slope of 1.091 ± 0.068, close to other CB slopes. Fitting the remaining $\omega({}^3S_1)$ trajectory with states at 782, 1425, and 2205 MeV, but with two missing states gives a slope of 1.080 ± 0.029. The fit to the second state however gives it a mass of 1329 MeV. Many authors have noticed this point. It could well be due to the fact that the ground state is abnormally low in mass, like the J/ψ and $\Upsilon(1S)$.

Section G considers f_0 states. Here physics remarks are required. One of the fundamentals of Particle Physics is Chiral Symmetry Breaking. Pioneering work on this topic was done by Bicudo and Ribiero [12], explaining how this arises. It accounts for the low mass $\sigma \equiv f_0(500)$, κ, $a_0(980)$ and $f_0(980)$. It is now well understood [13] how a crossover arises between these exceptional states and regular $q\bar{q}$ states near 1 GeV. The surviving mixing above 1 GeV is likely to push the $q\bar{q}$ 0^+ states up in mass. This can explain the anomalously high masses of $f_0(1370)$ and $a_0(1450)$. A further complication for f_0 is the likely existence of a glueball in the mass range 1500–1800 MeV, still obscure. Yet another complication is that the $f_0(1370)$ decays dominantly to 4π; this introduces large dispersive effects on the mass. It appears with a mass of 1309 MeV in $\pi\pi$ [14], but rapidly increasing phase space for its dominant $\rho\rho$ decay channel moves the peak in $\rho\rho$ up by ~ 75 MeV. Conclusions about the f_0 slope are therefore ambiguous. Using only the $\pi\pi$ mass of $f_0(1370)$ and the mass of $f_0(2330)$, the slope is 1.24 ± 0.045 GeV^{-2}.

A further point is that there is evidence [15] that the $f_2(1810)$ claimed by GAMS has been confused with the $f_0(1790)$ candidate for the radial excitation of $f_0(1370)$; the $f_0(1790)$ is consistent with the BES II 0^+ peak observed at 1812 MeV in $\omega\phi$ decays[15]. So, in summary, f_0 and f_2 states are complex. The $f_2(1640)$ has been explained away as the $\omega\omega$ decay mode of $f_2(1565)$; the rapidly rising $\omega\omega$ phase space shifts the peak in $\omega\omega$ up to 1640 MeV [16].

MAB construct two trajectories for what they take to be $n\bar{n}$ states and $s\bar{s}$. The $n\bar{n}$ trajectory starts with $f_0(980)$ and finishes with $f_0(2200)$. However, there is almost universal agreement today that $f_0(980)$ and $a_0(980)$ are not $q\bar{q}$ states but have dominant 4-quark composition [17]. The $f_0(980)$ decays dominantly to $K\bar{K}$, not $\pi\pi$. BES II quote a $KK/\pi\pi$ branching ratio $4.21 \pm 0.2(stat) \pm 0.21(syst)$ [18]; this is one of the very few experiments which has data on both

$K\bar{K}$ and $\pi\pi$. A further important source of information is the decay of $\chi^0 \to K^+K^-$. There is a conspicuous $f_0(1710)$ in the data and a further peak at $f_0(2200)$ [19]. This suggests that they both have substantial $s\bar{s}$ components. So the MAB $n\bar{n}$ trajectory looks unlikely.

On their $s\bar{s}$ trajectory, MAB start with $f_0(1370)$ and finish with $f_0(2330)$. The $f_0(1370)$ is observed dominantly in decays to 4π, largely $\rho\rho$. It has a branching ratio to $K\bar{K}$ of 0.12 ± 0.06 in CB data, so it does not look like an $s\bar{s}$ state. The last member of this trajectory is $f_0(2330)$. This has been observed in Crystal Barrel data in decays to $\pi\pi$ and $\eta\eta$ [8] with a flavour angle of 15.1°, so it is certainly not a dominantly $s\bar{s}$ state.

Section E of MAB discusses a_0, a_2 and a_4 states. This is again a complex story. They use the mass of $a_0(980)$, which is unwise in view of its association with Chiral Symmetry Breaking; it is now well established that it is not a simple $q\bar{q}$ state. The $a_0(1450)$ is well determined, but the signal for $a_0(2025)$ is very weak. An additional a_0 is possible somewhere between these two, but there are no data adequate to detect it; finding spin 0 states is difficult. Assuming this state has been missed, the slope from $a_0(1450)$ and $a_0(2025)$ is 0.964 ± 0.073 GeV^{-2}, but is probably affected by Chiral Symmetry Breaking and is not used in the overall CB average for the slope. MAB instead make the assumption that this is the third a_0 including $a_0(980)$ and hence find a slope of 1.42 ± 0.26.

Moving on to a_2 states, MAB consider as an upper trajectory ($^3F_2$) $a_2(2030)$ and $a_2(2255)$ and arrive at a similar slope to CB. For the $^3P_2$ trajectory, they take $a_2(1320)$, $a_2(1700)$ and $a_2(2175)$. This is to be compared with the well established f_2 trajectory $f_2(1270)$, $f_2(1565)$, $f_2(1910)$, $f_2(2240)$. They miss an a_2 state to be identified with the $a_2(1950)$ of Anisovich et al. [20]. They find a slope 1.39 ± 0.26 compared with the CB determination 1.00 ± 0.06.

There is agreement between CB and MAB for the a_4 slope. However, one comment is needed on the PDG determination of the mass. The PDG determined it largely from data of Uman et al. [21]. If one looks at Fig. 6 of Uman et al., the difference in χ^2 between a_2 and a_4 is small. They do not consider the possibility that both a_2 and a_4 are present; that would not be at all surprising. Therefore the CB determination of the mass is preferred here.

Consider next f_2 states. The problem here is that MAB do not discriminate between the $^3P_2$ states and $^3F_2$, which are well separated in CB data. MAB launch into four alternative scenarios, all of which have problems.

Their f_2^a trajectory is made from $f_2(1270)$, $f_2(1750)$ and $f_2(2150)$. The $f_2(2150)$ is not seen in CB data. It is observed only in decays to $K\bar{K}$ and $\eta\eta$. The $f_2(2010)$ of the PDG [3], observed by Etkin et al. in $\bar{p}p \to \phi\phi$ actually peaks at 2150 MeV. The mass quoted by Etkin et al. [22] is the K-matrix mass, and can differ from the T-matrix mass; the K-matrix formalism assumes that all decay channels are known, but that is unlikely. The obvious interpretation of the $f_2(2150)$ is the $s\bar{s}$ partner of $f_2(1910)$, i.e. a $^3P_2$ state. Etkin et al. also report an $f_2(2300)$ in the $\phi\phi$ S-wave and $f_2(2340)$ in the $\phi\phi$ D-wave. This is naturally to be interpreted as an $s\bar{s}$ $^3F_2$ state. The $f_2(1750)$ of Schegelsky et al. [23] is observed in $\gamma\gamma \to K\bar{K}$, and is interpreted by them as an $s\bar{s}$ state - the radial excitation of $f_2(1525)$, though rather low in mass.

The MAB f_2^b trajectory uses $f_2(1430)$. That entry in PDG tables has a straightforward interpretation. The $\omega\omega$ channel (and therefore $\rho\rho$) couples strongly to $f_2(1565)$. When analysing data on Dalitz plots, it is necessary to continue the Flatté formula for $f_2(1565)$ below the $\omega\omega$ threshold, rather than just cutting it off. This is the way in which Crystal Barrel analyses Dalitz plots. The result is a cusp in the $\pi\pi$ channel at 1430 MeV, see Fig. 7 of Adomeit et al. [24]. It is likely that the data listed by the PDG under $f_0(1430)$ were due to this phenomenon.

The f_2^c trajectory of MAB starts with $f_2(1525)$, which is a well known $s\bar{s}$ state and is obviously invalid. Their f_2^d trajectory uses $f_2(1565)$, $f_2(2000)$ and $f_2(2295)$, hence mixing $^3P_2$ and $^3F_2$ states. This is also invalid.

They continue with three further trajectories. The first is based on $f_2(1640)$, together with $f_2(2150)$. However, the $f_2(1640)$ has been explained by Baker et al. as the $\omega\omega$ decay of $f_2(1565)$. The second trajectory is based on the questionable $f_2(1810)$ and $f_2(2220)$, which is a very narrow peak, 23 MeV wide, claimed in BES II data. If such a narrow state contributes to non-strange $q\bar{q}$ states, it is a mystery why it is not observed very conspicuously in CB data. Their final trajectory is made of $f_2(2010)$ and $f_2(2340)$, which are observed in $\phi\phi$ and $K\bar{K}$; these are obvious candidates for $s\bar{s}$ states.

Section D of MAB concerns π and π_2 trajectories. These agree with CB. The $\pi(1800)$ is usually considered a hybrid; it has little effect on the fitted slope.

Section C discusses ρ_1 and ρ_3 states. Their slope for the latter is close to the CB value but with much larger error from their χ^2 criterion. The physics situation concerning ρ_1 states is a mess, for physics reasons. The $\rho(1450)$ couples weakly to 2π and there are large dispersive effects in the 4π channel, which have not yet been taken into account. The $\rho(1570)$ of Babar has a larger error in mass: $\pm 36(stat) \pm 62(syst)$ MeV and is marginally consistent with $\rho(1450)$, which actually has a mass of 1465 ± 25 MeV.

The $\rho(1900)$ can be identified with a recent Novosibirsk observation of a 6π peak almost exactly at the $\bar{p}p$ threshold [25]. This may be a $^3S_1$ state captured by the very strong $\bar{p}p$ S-wave, but could be a non-resonant cusp.

CB data list ratios r_J of coupling constants to orbital angular momentum $L = J + 1$ and $L = J - 1$. The $\rho(2000)$ has a sizable value $r_1 = 0.70 \pm 0.23$ requiring at least some $^3D_1$ contribution. The $\rho(2150)$ in CB data has $r_1 = -0.05 \pm 0.42$, consistent with $^3S_1$. The $\rho(2270)$ in CB data has $r_1 = -0.55 \pm 0.66$ which is ambiguous. However, the $\rho(2000), (2150), (2270)$ make a natural sequence of $^3D_1$, $^3S_1$, $^3D_1$. If this solution to the puzzle is accepted, the slope of the CB $^3S_1$ trajectory is 1.365 ± 0.108, with a rather poor fit to the mass of $\rho(1450)$. In view of the poor fit to $\rho(1450)$, this is not included in the grand average. The trajectory of $\rho(1700)$, $\rho(2000)$ and $\rho(2270)$ gives a slope of 1.20 ± 0.05; MAB quote 1.08 ± 0.47.

Section B of MAB discusses η and η_2 states. The $\eta(1760)$ and $\eta(2100)$ were claimed by DM2, but later identified in Mark III data [26] as having $J^{PC} = 0^{++}$, though they sit on a large non-interfering 0^{-+} background; [0^{++} and 0^{-+} do not interfere in J/ψ radiative decays after summing over relevant spin states of the J/ψ]. They are also identified in E760 data [27] in the $\eta\eta$ channel, where $J^{PC} = 0^{-+}$ is forbidden by Bose statistics. The remaining trajectory, $\eta(1295)$ and $\eta(2320)$ gives a CB slope of 1.24 ± 0.03; MAB quote 1.33 ± 0.11 because they include the other two states. A comment is that the mass of $\eta(2320)$ looks anomalously high compared with other higher spin states in this mass range.

Section A of MAB discusses the a_1 trajectory. The $a_1(2095)$ has a large error in mass of ± 121 MeV. The $a_1(2270)$ completes the trajectory. There is an obvious problem that the $a_1(1260)$ has a large width; the PDG quotes it as 250–660 MeV. A recent Babar estimate is $410 \pm 31 \pm 30$ MeV. The CB slope is 1.08 ± 0.06 GeV^{-2}; MAB find 1.42 ± 0.26 GeV^{-2}.

Finally, MAB discard all slope determinations which have only two points. This unfortunately removes all the determinations from $^3F_2$, $^3F_3$ and $^3F_4$ states which are amongst the best. As one sees from Table 1, these determinations have slopes consistent with other CB values. The $^3F_2$ state at 2293 MeV was identified in 5 channels of data in the year 2000 paper of Anisovich

et al. [28]. MAB also assign $a_2(2030)$ and $a_2(2255)$ the radial quantum numbers $n = 2$ and 3, while the nearby $f_2(2000)$ and $f_4(2295)$ obviously have $n = 3$ and 4.

In summary, the MAB classification of slopes unfortunately contains a number of problems, and there is no significant case for the large slopes they claim.

5 Epilogue

Values of CB and MAB differ significantly only where there are indications of problems in their selection of fitted states. The weighted mean of CB slopes is revised slightly. On close inspection, the χ^2 contributions from ${}^3S_1\,\rho_1$ results are high by a factor 4. Warnings about the problem in this case have already been given. Likewise contributions to χ^2 from a_0 are high by a factor 5. Again the text has pointed out problems for these states. Finally, χ^2 contributions from $a_2\ {}^3P_2$ states are high by a factor 4. This is no surprise, since there are no polarisation data to provide clear identifications of these states.

MAB remark that Anisovich, Anisovich and Sarantsev (AAS) proposed a scheme in 2002 where the lowest $J^{PC} = 1^{++}$ states were taken as $a_0(980)$ and $f_0(980)$ [29]. Since then, there have been many studies of the effects of Chiral Symmetry Breaking. It is now widely believed that the σ, κ, $a_0(980)$ and $f_0(980)$ are meson-meson states, and that there is a cross-over to $q\bar{q}$ near 1 GeV where Chiral Symmetry Breaking decreases rapidly. Weise presents a clear summary of these ideas [13]. This being the case, the AAS proposal no longer looks attractive.

Summarising, the mean slope derived from CB data without any corrections is 1.131 ± 0.011. Reducing the weights of the three troublesome cases to 1 modifies this to the final value 1.136 ± 0.012, compared with the old value of 1.143 ± 0.013 GeV^2. There is no clear case for the large error assessment of MAB. In fact, states with large widths already enlarge the errors for masses appropriately.

The PDG has decided to segregate these data on the grounds that they need confirmation. Perhaps, but $I = 0$, $C = +1$ contains a complete spectrum of expected states. For other isospin and C values, it is important to improve the data base. That cannot be done in production experiments, because the exchanged meson is not usually known, i.e. no polarisation information is available. The ρ and ω states can be improved at VEPP 2 in Novosibirsk by using transversely polarised electrons. Two measurements are readily made of asymmetries normal to the plane of polarisation and in the plane of polarisation. Electron polarisation of 70% is already achieved and two detectors CMD and SMD are available and running. The presence of ${}^3D_1$ states is then revealed by distinctive azimuthal angular dependence in the polarisation and can measure whether these are pure ${}^3D_1$ states or linear combinations with ${}^3S_1$, and if so how big the contributions are. Longitudinal polarisation does not help much because it depends only on the difference of intensities of the two helicities available.

In order to trace the missing $n\bar{n}$ states above 1910 MeV, polarisation measurements are needed for $I = 1$ $C = +1$ ($\eta\eta\pi^0$, $\eta\pi^0$ and $3\pi^0$), $I = 1$, $C = -1$ ($\omega\pi^0$ and $\eta\omega\pi^0$), and $I = 0$, $C = -1$ ($\omega\eta$ and $\omega\pi^0\pi^0$). Polarisation data also introduce interference between singlet and triplet states, hence determining the singlet states much better. The Crystal Barrel detector is still running in Bonn and has a polarised target ideal for these measurements. From experience in earlier polarisation experiments and Monte Carlo simulations, average backgrounds from heavy nuclei in a NH_3 target are $\sim 8\%$ after event reconstruction and this is adequate; it may result in some

blurring of any deep dips in angular distributions but that can be corrected. At LEAR, the data handling system could handle only 60 events/s, but this has already been upgraded to 1000/s or more. The upper limit on $\bar{p}$ beam intensity is $\sim 5 \times 10^4/s$ because of pile-up in the CsI crystals. At LEAR, the approved program forsaw momenta of 750, 500 and 360 MeV/c in order to cover both sides of the tower of resonances peaking near a mass of 2000 MeV. This was cut short by the premature closure of LEAR. Differential cross sections are needed as well as polarisation measurements at these 3 low momenta and 600 MeV/c, where statistics are low; this can be done with a liquid hydrogen target. All required software for event reconstruction and partial wave analysis is already available.

PANDA cannot do this measurement because the ring planned for it has a lower limit of 1.5 GeV/c. A stopping $\bar{p}$ beam is already forseen for experiments on CPT violation in atomic spectra. The one additional requirement is a slow extraction for low momentum $\bar{p}$, using the technology developed at LEAR. That is highly desirable anyway. The required beam time to run the polarisation measurements is ~ 4 months of beam-time, but doubtless distributed over a longer calendar time. The present program at ELSA should be run to its natural conclusion first, but then the cost of moving the detector to FAIR is small. A minor modification of the target would allow completion of the study of $\bar{p}p \to \Lambda\bar{\Lambda}$ and $\Sigma\bar{\Sigma}$.

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